

B. L. R. SHAWYER AND B. B. WATSON, *Borel's Methods of Summability*, Oxford Mathematical Monographs, Oxford Science Publications, 1994, xii + 242 pp.

This book deals with a family of summability methods closely related to the famous method of Borel. We say that a complex sequence  $(s_n)$  converges to  $s$  in the sense of Borel, in short  $s_n \rightarrow s (B)$ , if  $p_s(x) = \sum s_k (x^k/k!)$  converges for all  $x \in \mathbb{R}$ , and  $\sigma(x) := e^{-x} p_s(x) \rightarrow s$  as  $x \rightarrow \infty$ . This is a more general notion of convergence. In particular if  $(s_n)$  converges to  $s$  in the ordinary sense then  $s_n \rightarrow s (B)$ . This method is well understood and is relevant to many problems in mathematics.

This book deals with the following topics. After a nice historical overview (Section 1), the authors introduce in Section 2 the notion of summability methods and illuminate it by discussing three families of methods: Cesàro, power series, and Euler methods. In Sections 3 and 4 a collection of well-known classical results about Borel summability, and relations to other methods such as Cesàro and Euler methods, and then in particular the circle methods are presented. To this point the results are presented mostly without proofs. Starting with Section 5 the authors consider a natural generalization of Borel's method, the so-called Borel-type summability methods  $B(\alpha, \beta)$ , where the transformation of  $(s_n)$  is given by

$$\sigma(x) = \alpha e^{-x} \sum_{n=N}^{\infty} s_n x^{2n+\beta-1} / \Gamma(\alpha n + \beta) \quad \alpha > 0, \beta \in \mathbb{R}, N \in \mathbb{N}, \alpha N + \beta > 0.$$

From this point on proofs are generally included. After a discussion of these methods, the book deals with the interrelationship between  $B(\alpha, \beta)$ -methods for different  $\alpha, \beta$  (Section 5), Abelian theorems (Section 6), Tauberian theorems—local Tauberian conditions (Section 7), oscillation conditions and the deep gap-Tauberian theorem (Section 8). However, with respect to the latter no proof is provided. Finally, in Section 9 the relationship with other methods is investigated. In the last section some applications to the field of entire functions and arithmetical functions are presented.

The book is a collection of many interesting results related to Borel-type methods which are scattered throughout the literature. It includes an extensive bibliography which is a good source for relevant papers. The text is well written and I enjoyed reading it. There are only a few points which could be criticized. Some of the more recent results in the field are not included, e.g., the Tauberian theorems are true for a much more general class of summability methods, without being more complicated (results of D. Borwein, W. Kratz, and the reviewer). Some interesting relations to other fields in mathematics are not mentioned, e.g., for the asymptotics some probability is useful and natural (local central limit theorems). The applications range from probability to physics and some discussion of these would have made the book more interesting for the nonspecialist.

ULRICH STADTMÜLLER

G. E. ANDREWS, B. C. BERNDT, L. JACOBSEN, AND R. L. LAMPHERE, *The Continued Fractions Found in the Unorganized Portions of Ramanujan's Notebooks*, Memoirs of the American Mathematical Society **477**, Amer. Math. Soc., Providence, RI, 1992, vi + 71 pp.

The authors discuss 60 continued fraction entries in 133 pages of the unorganized portions of Ramanujan's second and third notebooks (published in two volumes by the Tata Institute of Fundamental Research, Bombay, 1957). Many of these entries are related to previous entries in the organized portions of the second notebook, especially Chapters 12 and 16 which were examined by Berndt, Lamphere, and Wilson [*Chapter 12 of Ramanujan's second notebook: continued fractions*, *Rocky Mountain J. Math.* **15** (1985), 235–310] and by Adiga, Berndt, Bhargava, and Watson [*Chapter 16 of Ramanujan's notebook: theta-functions and q-series*,